A biography of Hillel Furstenberg

When Hillel (Harry) Furstenberg published one of his early papers, a rumour circulated that he was not an individual but instead a pseudonym for a group of mathematicians. The paper contained ideas from so many different areas, surely it could not possibly be the work of one man?

The anecdote may be apocryphal but it illustrates a truth that has been consolidated throughout his academic life: Furstenberg has a deep technical knowledge of diverse areas and has made insightful and surprising connections between them. In particular, he has made fundamental contributions to the field of ergodic theory, which have had far-reaching applications in number theory, geometry, combinatorics, group theory and probability.

Furstenberg was born in Berlin in 1935. His family was Jewish. A few months before the outbreak of the Second World War, they managed to get out of Germany and fled to the U.S. Furstenberg's father died on the journey, and Hillel was brought up by his mother and elder sister in an orthodox community in New York.

Furstenberg became interested in mathematics when he saw his teacher getting into a muddle when explaining well-known theorems. The student enjoyed finding his own proofs. "Sometimes bad teachers do a good job!" he says. He attended the high school and college of Yeshiva University, graduating in 1955 with a B.A. and an M.S. As an undergraduate he was already publishing papers. *Note on one type of indeterminate form* (1953) and *On the infinitude of primes* (1955) both appeared in the *American Mathematical Monthly*, the latter giving a topological proof of Euclid's famous theorem that there are infinitely many primes.

Furstenberg went to Princeton University to study for his doctorate under the supervision of Salomon Bochner. He received his PhD in 1958 for his thesis, *Prediction Theory*. When it was published in 1960

one reviewer said: "the work stands as a first-rate and highly original dissertation on a very difficult subject".

After spending a year as an instructor at Princeton and one at MIT, he got his first job as an assistant professor in 1961 at the University of Minnesota. In a series of articles beginning in 1963 with *A Poisson Formula for Semi-Simple Lie Groups*, he continued to establish himself as a ground-breaking thinker. His work showing that the behaviour of random walks on a group is intricately related to the structure of the group — which led to what is now called the 'Furstenberg Boundary' — has been hugely influential in the study of lattices and Lie groups.

He was promoted to full professor at Minnesota, but in 1965 he left the U.S. for the Hebrew University of Jerusalem where he stayed until his retirement in 2003.

In his 1967 paper, *Disjointness in ergodic theory, minimal sets, and a problem in Diophantine approximation*, Furstenberg introduced the notion of "disjointness", a notion in ergodic systems that is analogous to coprimality for integers. The notion turned out to have applications in areas such as number theory, fractals, signal processing and electrical engineering.

In his 1977 paper, *Ergodic behavior of diagonal measures and a theorem of Szemerédi on arithmetic progressions*, Furstenberg used methods from ergodic theory to prove a celebrated result by Endre Szemerédi (Abel Prize laureate 2012), which states that any subset of the integers with positive upper density contains arbitrarily large arithmetic progressions. Furstenberg's proof was more conceptual than Szemerédi's and it completely changed the area. Its insights also became very fruitful, leading to important results such as the proof by Ben Green and Terence Tao that the sequence of prime numbers includes arbitrary large arithmetic progressions.

Furstenberg's decision to spend almost all of his career in Israel helped establish the country as a world centre for mathematics, in particular for ergodic theory. In the academic year 1975/76, he ran a year-long programme on ergodic theory at the Israeli Institute of Advanced Study, together with Benjamin Weiss, which is considered to have transformed the field.

Of his many accolades, Furstenberg has won the Israel Prize, regarded as the top honour in Israel, and the Wolf Prize in mathematics. He is a member of the Israel Academy and the American Academy of Arts and Science.

Furstenberg married Rochelle, a magazine writer specialising in arts and culture, in 1958. They have five children, sixteen grandchildren, and a growing number of great-grandchildren.