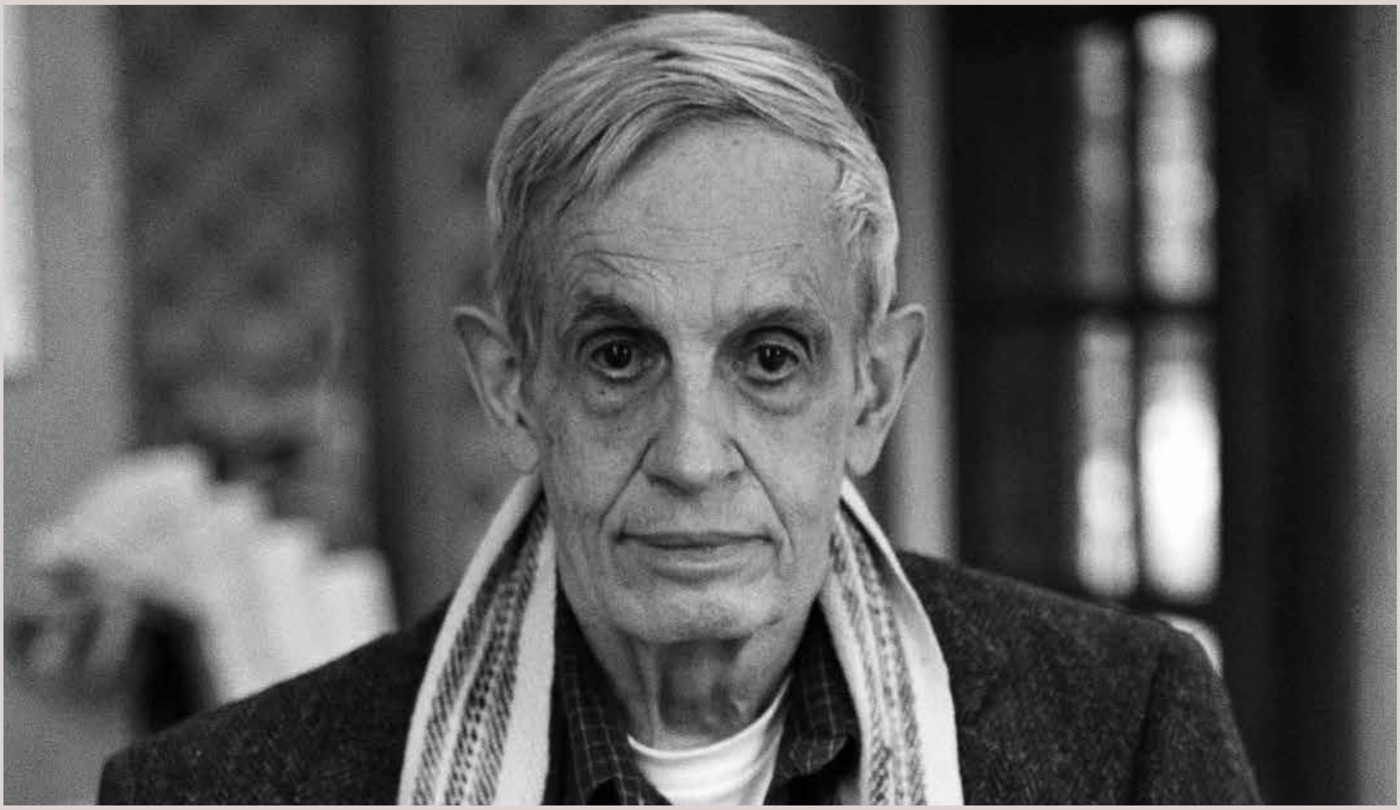

John Forbes Nash Jr.

John F. Nash Jr. is one of a handful of mathematicians known outside academia, due to the 2001 film about him, *A Beautiful Mind*, loosely based on Sylvia Nasar's bestselling biography of the same name. The Oscar-winning movie fictionalized Nash's path from brilliant Princeton student to being awarded the 1994 Nobel Prize for economics.

Inevitably, the Hollywood version of Nash's life story differed from the real one in many ways. In particular, the film focused on his early results in game theory, which have applications in economics, and omitted his research into geometry and partial differential equations, which the mathematical community regards as his most important and deepest work.

John Forbes Nash Jr. was born in 1928 in Bluefield, West Virginia, a small, remote town in the Appalachians. His father was an electrical engineer at the local power company and his mother a schoolteacher. He entered the Carnegie Institute of Technology (now Carnegie Mellon University) in Pittsburgh with a full scholarship, originally studying for a major in chemical engineering, before switching to chemistry and finally changing again to mathematics.

At Carnegie, Nash took an elective course in economics, which gave him the idea for his first paper, *The Bargaining Problem*, which he wrote in his second term as a graduate student at Princeton University. This paper led to his interest in the new field of game theory – the mathematics of decision-making. Nash's Ph.D. thesis, *Non-Cooperative Games*, is one of the foundational texts of game theory. It introduced the concept of an equilibrium for non-cooperative games, the "Nash equilibrium", which has had a great impact in economics and the social sciences.

While at Princeton Nash also made his first breakthrough in pure mathematics. He described it as "a nice discovery relating to manifolds and real algebraic varieties." In essence the theorem shows that any manifold, a topological object like a surface, can be described by an algebraic variety, a geometric object defined by equations, in a much more concise way than had previously been thought possible. The result was already regarded by his peers as an important and remarkable work.

In 1951 Nash left Princeton to take up an instructorship at MIT. Here he became interested in the Riemann embedding problem, which asks whether it is possible to embed a manifold with specific rules about distance in some *n*-dimensional Euclidean space such that these rules are maintained. Nash provided two theorems that proved it was true: the first when smoothness was ignored and the second in a setting that maintained smoothness.

In order to prove his second embedding theorem, Nash needed to solve sets of partial differential equations that hitherto had been considered impossible to solve. He devised an iterative technique, which was then modified by Jürgen Moser, and is now known as the Nash–Moser theorem. The Abel Prize laureate Mikhail Gromov has said: “What [Nash] has done in geometry is, from my point of view, incomparably greater than what he has done in economics, by many orders of magnitude. It was an incredible change in attitude of how you think about manifolds. You can take them in your bare hands, and what you do may be much more powerful than what you can do by traditional means.”

In the early 1950s Nash worked as a consultant for the RAND Corporation, a civilian think-tank funded by the military in Santa Monica, California. He spent a few summers there, where his work on game theory found applications in United States’ military and diplomatic strategy.

Nash won one of the first Sloan Fellowships in 1956 and chose to take a year’s sabbatical at the Institute of Advanced Study in Princeton. He based himself not in Princeton, but in New York, where he spent much of his time at Richard Courant’s fledgling Institute for Applied Mathematics at NYU. It was here Nash met Louis Nirenberg, who suggested to him that he work on a major open problem in nonlinear theory concerning inequalities associated with elliptic partial differential equations. Within a few months Nash had proved the existence of these inequalities. Unknown to him, the Italian mathematician Ennio De Giorgi had already proved this, using a different method, and the result is known as the Nash-De Giorgi theorem.

Nash was not a specialist. He worked on his own, and relished tackling famous open problems, often coming up with completely new ways of thinking. In 2002 Louis Nirenberg said: “About twenty years ago somebody asked me, ‘Were there any mathematicians you would consider as geniuses?’ I said, ‘I can think of one, and that’s John Nash.’… He had a remarkable mind. He thought about things differently from other people.”

In 1957 Nash married Alicia Larde, a physics major whom he met at MIT. In 1959 when Alicia was pregnant with their son, he began to suffer from delusions and extreme paranoia and as a result resigned from the MIT faculty. For the next three decades Nash was only able to do serious mathematical research in brief periods of lucidity. He improved gradually and by the 1990s his mental state had recovered.

The 1990s also saw him receive a number of honours for his professional work. As well as winning the prize in economic sciences in memory of Alfred Nobel in 1994, which he shared with John C. Harsanyi and Reinhard Selten, he was elected a member of the National Academy of Sciences in 1996, and in 1999 he won the American Mathematical Society’s Steele Prize for Seminal Contribution to Research for his 1956 embedding theorem, sharing it with Michael G. Crandall.